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Some Nonintuitive Features in Time-Efficient Attitude Maneuvers of Combat Aircraft

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In some cases, numerical extremal trajectories for time-optimal attitude maneuvers for combat aircraft exhibit a nonintuitive feature in which the extremal roll-control causes the vehicle to initially roll the "wrong way." Using data for the high angle-of-attack research vehicle, we present a brief exposition and engineering explanation of this and related phenomena. The explanations are based on analyses of the structure of the aircraft mathematical model.

I. Introduction

To utilize an aircraft most effectively in a combat situation, the pilot needs to perform tactical maneuvers in the most efficient way, in an optimal manner. Modern computers and advanced numerical techniques, along with accurate aerodynamic models of aircraft obtained from wind-tunnel measurements, facilitate numerical study of the problem of optimal tactical maneuvering.[1]

Because of the complexity of the aircraft-environment dynamic system, the search for extremal trajectories poses difficult numerical problems. These can be treated effectively if an interactive approach is adopted: qualitative analyses of the dynamic system and the extremal trajectory sought (i.e., relating numerical problems to the physics of the dynamic system) can help us guide the numerical procedures efficiently.

The authors have recently reported[2,3] an analytical mathematical model for the high angle-of-attack research vehicle (HARV), developed for study of time-optimal attitude maneuvers. The model neglects the translational motion of the aircraft, and thus is valid only for rapid attitude maneuvers, during which the aircraft velocity-vector does not change significantly. Numerical extremal trajectories for several important classes of attitude-reorientation maneuvers have been found, using optimal control theory,[4] and studied thoroughly. In some cases, the extremal trajectories exhibit a somewhat unexpected, nonintuitive feature in which the extremal roll-control causes the vehicle to roll the wrong way at the initial and/or the final portion of the trajectory. In this paper we examine this wrong-direction roll phenomenon through analysis of a family of extremal trajectories for the class of "roll around the velocity-vector" (RVV) maneuvers.

The mathematical model of the HARV is presented in Sec. II. The structure of the dynamic system (mathematical model) is emphasized, and a few relevant levels of dynamic coupling are identified. The class of RVV attitude maneuvers is defined in Sec. III, and a family of regular extremal trajectories for time-optimal 90 deg-RVV maneuvers is described. A case study is presented about one particular extremal trajectory of the 90 deg-RVV family, and the wrong-direction roll phenomenon is explained.

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II. Aircraft Model for Attitude Maneuvers

The mathematical model used in this study is briefly presented later. To simplify the discussion, in this paper we confine our attention to a model of the HARV with no thrust-vectoring capability. A detailed derivation and description of the mathematical model can be found in Refs. 2 and 3.

The interest in fast and large-attitude maneuvers is restricted to low values of Mach number and dynamic pressure.[5] Rapid attitude maneuvers at high dynamic pressure are likely to be unacceptable for the human pilot, due to the high side accelerations developed. At moderate altitudes (say $h \approx 15{,}000$ ft) and low Mach numbers ($M < 0.4$), for a broad class of rapid fuselage-reorientation maneuvers, the velocity-vector of the aircraft does not change significantly in the course of the maneuver. Namely, the motions are fast enough[3] (on the order of 1 or 2 s) that the linear displacements of the aircraft center-of-mass from the line of the original direction and the changes in Mach number and dynamic pressure are small. These facts allow us to neglect the linear motion of the aircraft in the course of the attitude maneuvers. One can visualize the aircraft in a constant-velocity wind tunnel, free to rotate about the center-of-mass. Basically, we are interested in how the aerodynamic control surfaces should be utilized so as to reorient the aircraft from one attitude to another in minimum time. State variables in our model are the angular rates p, q, and r, and the attitude angles α, β, and μ. Because of the assumption of constant velocity-vector, the angles α, β, and μ determine the aircraft attitude uniquely (μ is the wind-axes roll angle[6]).

The aircraft aerodynamic moments are very complex functions. We assume that the (differential) ailerons (Δ_a), horizontal tails (δ_e), and the rudders (δ_r) can be used only for roll, pitch, and yaw control, respectively. Thus, the vehicle dynamics can be modeled plausibly if the following functional dependencies are assumed:

$$C_l = C_l^0(\alpha,\beta) + C_l^{\zeta}(\alpha,p) + C_l^c(\alpha,\Delta_a) \tag{1a}$$

$$C_m = C_m^0(\alpha,\beta) + C_m^{\zeta}(\alpha,q) + C_m^c(\alpha,\delta_e) \tag{1b}$$

$$C_n = C_n^0(\alpha,\beta) + C_n^{\zeta}(\alpha,r) + C_n^c(\alpha,\delta_r) \tag{1c}$$

Here C_i^0, C_i^{ζ}, and C_i^c ($i \in \{l,m,n\}$) denote the rigid-body static (all control surfaces in neutral position), rate damping, and aerodynamic control-surface contributions to the aerodynamic moment coefficients, respectively.

A. Scaled Mathematical Model

The angular rates and the time variable are scaled for numerical reasons. The scaled mathematical model is

$$\dot{\alpha} = q - (p\cos\alpha + r\sin\alpha)\tan\beta \tag{2a}$$

$$\dot{\beta} = p \sin \alpha - r \cos \alpha \tag{2b}$$

$$\dot{\mu} = (p \cos \alpha + r \sin \alpha)/\cos \beta \tag{2c}$$

$$\dot{P} = \underbrace{J_x qr}_{L_{qr}} + \underbrace{\mathcal{L} C_l(\alpha,\beta,p,\Delta_a)}_{L_a} \tag{3a}$$

$$\dot{Q} = \underbrace{J_y rp}_{M_{rp}} + \underbrace{\mathfrak{M} C_m(\alpha,\beta,q,\delta_e)}_{M_a} \tag{3b}$$

$$\dot{R} = \underbrace{J_z pq}_{N_{pq}} + \underbrace{\mathfrak{N} C_n(\alpha,\beta,r,\delta_r)}_{N_a} \tag{3c}$$

where [compare to Eqs. (1a–1c)]

$$L_a = \underbrace{\mathcal{L} C_l^0}_{L_0} + \underbrace{\epsilon_\zeta \mathcal{L} C_l^\zeta}_{L_\zeta} + \underbrace{\epsilon_a \mathcal{L} A(\alpha)\Delta_a}_{L_c} \tag{4a}$$

$$M_a = \underbrace{\mathfrak{M} C_m^0}_{M_0} + \underbrace{\epsilon_\zeta \mathfrak{M} C_m^\zeta}_{M_\zeta} + \underbrace{\epsilon_e \mathfrak{M}\left[E(\alpha)\delta_e + F(\alpha)\right]}_{M_c} \tag{4b}$$

$$N_a = \underbrace{\mathfrak{N} C_n^0}_{N_0} + \underbrace{\epsilon_\zeta \mathfrak{N} C_n^\zeta}_{N_\zeta} + \underbrace{\epsilon_r \mathfrak{N} R(\alpha)\delta_r}_{N_c} \tag{4c}$$

The state vector is $x = (\alpha, \beta, \mu, p, q, r)^T$. The control variables Δ_a, δ_e, and δ_r are normalized, so that each of them can vary in the interval $[-1, +1]$. The ϵ-variables will be referred to as homotopy parameters; the mathematical model represents the physical system of interest (the HARV) when each of the homotopy parameters has a unit value. The other parameters appearing in the model are constants, related to the aircraft design parameters and flight conditions (e.g., $J_x = (I_y - I_z)/I_x$, $\mathcal{L} = \rho V^2 Sb/2I_x$, etc).

B. Optimality Condition

We are interested in steering the system from a given initial state to a given final state, in minimum time. No state constraints are imposed. Normality[4] of the problem is assumed ($\lambda^0 = -1 < 0$). The aerodynamic controls appear linearly in the system dynamics and are independent. For regular (nonsingular) extremal trajectories the optimality condition[4] yields

$$\Delta_a = \mathrm{sgn}\left[\lambda_p \mathcal{L} A(\alpha)\right] = \mathrm{sgn}(\lambda_p) \tag{5a}$$

$$\delta_e = \mathrm{sgn}\left[\lambda_q \mathfrak{M} E(\alpha)\right] = \mathrm{sgn}(\lambda_q) \tag{5b}$$

$$\delta_r = \mathrm{sgn}\left[\lambda_r \mathfrak{N} R(\alpha)\right] = \mathrm{sgn}(\lambda_r) \tag{5c}$$

Complete derivation of the optimality conditions can be found in Ref. 2.

C. Levels of Dynamic Coupling

The following qualitative levels of dynamic coupling in the mathematical model can be identified:

1) The attitude angles α and β affect the dynamics of the angular rates P, Q, and R in a rather complicated manner (through the aerodynamic moment coefficients).

2) The angular rates affect the dynamics of the attitude angles (they can be thought of as controls in the attitude-angle channels).

3) The angular rates mutually affect the angular-rate dynamics through the gyroscopic terms and are self-influenced through the rate-damping terms.

III. 90 deg-RVV Extremal Family

In the rest of the paper we look at one particular class of attitude maneuvers: the 90 deg-RVV maneuvers, defined by the following initial and final state vectors:

$$x_0 = (\alpha_{0f}, 0, 0, 0, 0, 0)^T \tag{6a}$$

$$x_f = (\alpha_{0f}, 0, +90 \text{ deg}, 0, 0, 0)^T \tag{6b}$$

The initial and final angle of attack are identical, and the vehicle effectively rolls 90 deg around the velocity-vector.

The RVV maneuvers are of practical interest, and their execution in a time-optimal manner has considerable tactical value. They can be performed by rolling and yawing the aircraft while maintaining the sideslip angle zero (or close to zero) and the angle-of-attack (close to) constant. However, simulator studies have shown that by unloading the aircraft (pitching down to lower angles of attack), maneuvering time can be improved. A family of extremal trajectories resembling this motion is found, and it is the topic of our further discussion and analysis. A parameter of the extremal family is the initial and final angle of attack α_{0f}. Each trajectory of the family can be characterized as being a "pitch-down—pitch-up" sequence, while simultaneously performing roll and yaw motion (both in a positive sense). This characteristic motion of the extremal family facilitates development of relatively small sideslip angles in the course of the maneuver.[7] A number of the extremal trajectories, however, slightly deviate in the roll time history: they have an interval of wrong-direction roll motion at the beginning of the trajectory (and some trajectories at the end, as well), apparently caused by extremal aileron action in the wrong direction.

We examine further the 90 deg-RVV trajectory with $\alpha_{0f} = 30$ deg, state time histories for which are shown in Figs. 1 and 2. The aerodynamic controls time history, and the variational adjoint variables λ_p, λ_q, and λ_r, are shown in Fig. 3. We will refer to this trajectory as the nominal one and denote the

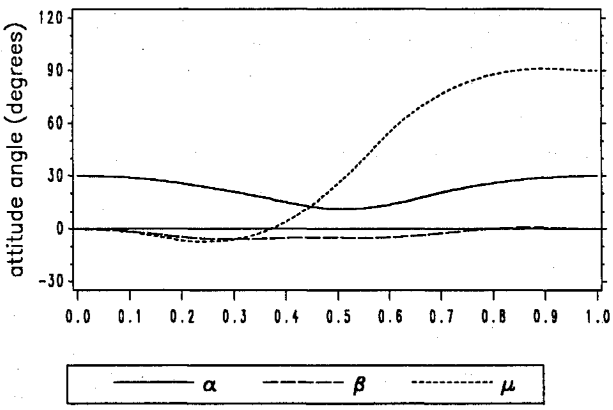

Fig. 1 Attitude angles time history.

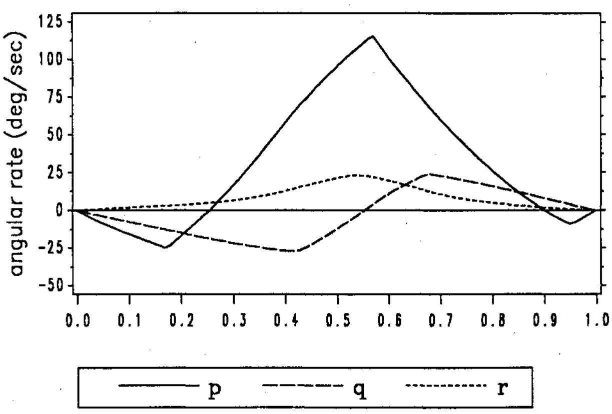

Fig. 2 Angular rates time history.

corresponding maneuvering time by T_n. The horizontal axes in these plots represent the normalized time $\tau = t/T_n$.

A. Dynamic Aspects

As a consequence of relatively small sideslip angles being developed by the extremal trajectory (see Fig. 1), the gross effect of the rigid-body static components L_0 [Eq. (4a)] and N_0 [Eq. (4c)] on maneuvering time is negligible.

By nature, the rate-damping components L_ζ, M_ζ, and N_ζ [Eqs. (4a–4c)], oppose the aerodynamic control-action in the rate-accelerating portion of the trajectory and act supportively in the rate-decelerating portion. Hence, rate damping prolongs the accelerating portion of the trajectory, shortens the decelerating portion, and is the sole cause of the nonsymmetry in the trajectory (with respect to $\tau = 0.5$; see Fig. 2). However, the total effect of rate damping on maneuvering time is relatively small. If rate damping did not physically exist [this can be simulated numerically by setting the rate-damping homotopy parameter to zero, $\epsilon_\zeta = 0$ Eqs. (4a–4c)] maneuvering time would have been only $\approx 0.154\% T_n$ faster.

The gyroscopic components L_{qr}, M_{rp}, and N_{pq} provide state coupling within the dynamic subsystem (3a–3c). Although the gyroscopic component L_{qr} is insignificant in the roll channel, in the pitch channel M_{rp} is of the same order of magnitude as the aerodynamic component M_a, and N_{pq} is the predominant component in the yaw channel, in the course of the trajectory.[3] Each of L_{qr}, M_{rp}, and N_{pq} supports well the action of the aerodynamic control in the corresponding channel, which is a highly desirable feature: we can think of the gyroscopic moments as being a "supportive addition" to the aerodynamic controls, increasing their effectiveness, and thus improving maneuvering time. Such supportive behavior of the gyroscopic terms is a consequence of the character of the extremal-family motion.

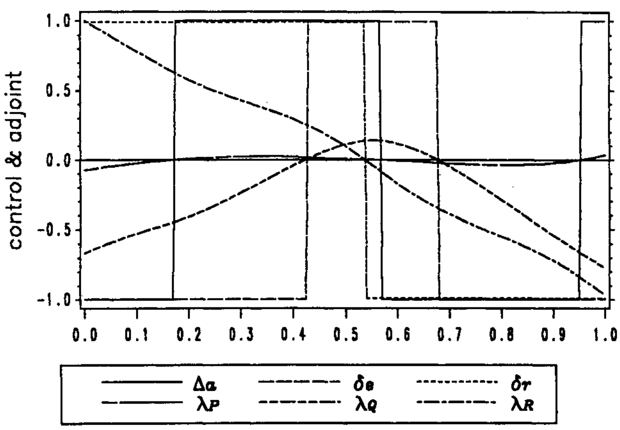

Fig. 3 Aerodynamic controls and rate adjoints.

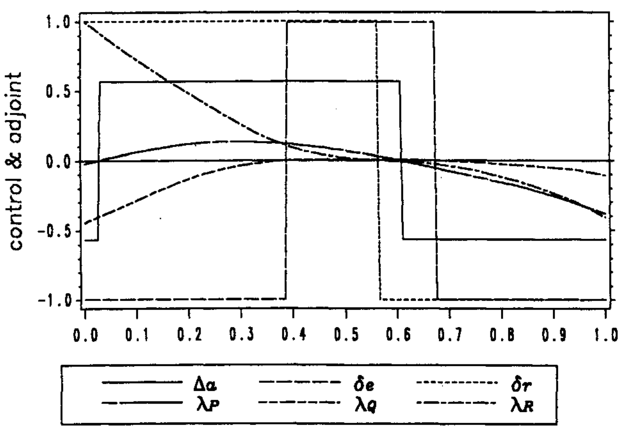

Fig. 4 Aerodynamic controls and rate adjoints (reduced aileron power).

The characteristic motion of this family also facilitates lower angles of attack (in the central portion of the trajectory), where the aerodynamic control surfaces are more effective. The aileron is the most powerful control, and the rudder is the least powerful one (by design).

B. Aerodynamic Roll-Control

The following numerical experiment is instructive for understanding the control balance and the aileron wrong-direction actions. First, we generate a series of extremals by fictively decreasing the maximal aileron control-power (by decreasing the aileron homotopy parameter ϵ_a, starting from 1). Analysis of the evolution of thus obtained extremal trajectories shows that these retain the character of the extremal-family motion. There is slight change in the relative duration of the positive and negative action intervals of the rudder and aileron. Most remarkable, as ϵ_a decreases from 1.0 to 0.565, is the evolution of the aileron time history. At first, the duration of the ending interval of wrong-direction action (about x_f) decreases gradually, and slightly below $\epsilon_a = 0.750$ it disappears. As ϵ_a decreases further to $\epsilon_a = 0.565$, the initial interval of wrong-direction action (about x_f) decreases steadily, and for $\epsilon_a = 0.565$ it lasts $\approx 2.67\%$ (see Fig. 4). Most amazingly, as ϵ_a is decreased from 1.0 to 0.565 (the aileron effectiveness decreased almost 50%), maneuvering time decreases only $\approx 4.86\% T_n$.

Next, we generate a series of extremal trajectories for fictively increased maximal aileron control-power. As ϵ_a increases from 1, the aileron wrong-direction action intervals become longer. For $\epsilon_a = 4$, these intervals are each $\approx 20\% T_{\epsilon_a = 4}$. The roll rate becomes strikingly large in the central portion of the trajectory, and the wrong-direction roll rates are quite substantial. The positive elevator action interval in the central portion of the trajectory decreases to $\approx 10\% T_{\epsilon_a = 4}$. Since the intervals of major positive and negative aileron action are also decreased (to a total of $\approx 60\% T_{\epsilon_a = 4}$), the positive action of the gyroscope M_{rp}-term is (relatively) narrower than in the nominal trajectory. Thus, for a greater portion of the maneuvering time the elevator inserts negative action in the pitch channel, and the gyroscopic component effectively becomes the primary generator of positive action. The pitch curve $q(\tau)$ retains its shape, and the maximum pitch rates are only slightly lower than in the nominal extremal. The $r(\tau)$ curve is close to zero in the intervals from $\tau = 0$ to $\tau \approx 0.35$ and from $\tau \approx 0.65$ to $\tau = 1$ (the N_c and N_{pq} components practically annihilate each other in those intervals). Thus, (positive) yaw rates and certain yaw motion take place only in the central portion of the trajectory, where almost 180 deg-roll motion takes place simultaneously.

C. Kinematic Aspects

From the previous discussion, it becomes clear that the wrong-direction roll (Fig. 3) at the beginning and at the end of our trajectory is related to relative excess of roll control-power. Namely, given a maneuver specified by the initial and the final state [Eqs. (6a) and (6b)], and given the corresponding trajectory of our extremal family inherent one can associate a certain amount of roll motion, pitch motion, and yaw motion. We can think of the aileron as being relatively more potent than the elevator and the rudder, relative with respect to the need of the extremal trajectory for roll, pitch, and yaw motion. As stated in Sec. II.C, the angular rates can be considered as controls for the kinematic subsystem [obviously, the p, q, and r time histories (see Fig. 2) control the amount of roll, pitch, and yaw motion that takes place (Fig. 1)]. In an extremal trajectory, the optimality condition (4a–4c) demands that the aerodynamic controls be used with full power (bang-bang), to steer the system from x_0 to x_f. Sometimes this is impossible, and the controls may need to be used with less than full power (i.e., in a singular manner[8]) along certain arcs of the trajectory (singular arcs). The question of existence of singular extremal trajectories for the 90 deg-RVV maneuvers is beyond the scope of this paper. However, it is clear that in our extremal trajectories the aileron's relative excess of control-power is effectively counterbalanced by a wrong-direction action at the initial and final segment of the trajectory. An interesting question arises as to whether and how this is beneficial from the time-optimality standpoint.

The previous results can be understood from yet another perspective. Since capability of yaw-rate generation is inferior to the achievable pitch and roll rates, we could expect that time-optimal extremal trajectories (besides developing in such a manner that the inferior yaw channel gets much support through gyroscopic state-coupling) develop in such a manner as to economize on yaw motion. This further elucidates the state-coupling tradeoffs between the kinematic subsystem (2a-2c) and the dynamic subsystem (3a-3c).

If the yaw channel were not capable of generating yaw motion at all, (that is, the yaw dynamics being $\dot{R} = 0$), a way of performing a 90-deg-RVV maneuver is by executing the following sequence of motions:

1) pitch-down ($\alpha_{0f} \mapsto 0$ deg)

2) roll (+ 90 deg) (7)

3) pitch-up (0 deg $\mapsto \alpha_{0f}$)

Kinematically, the pitch-down—roll—pitch-up motion sequence can substitute for the lack of yaw-motion capability. If we also request a time-optimal maneuver, the pitch and the yaw motion should be performed simultaneously, but the time-optimal trajectory will still resemble the motion from sequence (7) (and indeed our characteristic family motion does).

In Sec. III.B we saw what happens when the aircraft possesses much more roll power than the nominal model. The trajectory encountered for large ϵ_a resembles the following sequential motion:

1) ($-\phi_{\text{roll}}$) 2) ($-\theta_{\text{pitch}}$)

3) roll (+180 deg) (8)

4) ($+\theta_{\text{pitch}}$) 5) ($-\phi_{\text{roll}}$)

Besides sequence (7), this is another way for a 90 deg-RVV maneuver to be performed in a lack of yaw-motion generation capability.

D. Characteristic Extremal-Family Motion

At this point we understand that there is advantage in the relative excess of aileron control-power being counterbalanced (compensated) by wrong-direction action at the beginning and at the end of the trajectory. One can imagine why this excess is advantageous when compared to singular behavior or through chattering arcs. The wrong-direction roll is a right one, indeed it is just a way of redistributing the need for dynamic action in the angular-rate channels (indirectly, through the attitude-angle channels). The wrong-direction roll at the initial and final point is a means for the aircraft to perform more roll motion, and less yaw motion in the course of the maneuver. Thus, the apparent relative excess of roll power, or equivalently, the relative deficiency of yaw power, is appropriately compensated for, and the full power bang-bang controls steer our system precisely from x_0 to x_f. We can pronounce the character of our extremal-family motion as hybrid, incorporating characteristics of both the (7) and (8) motion sequences.

The wrong-direction roll in our extremal family illustrates how apparently irrational behavior of the trajectory on a local level can actually be a means for enabling the system to take advantage of various levels of dynamic coupling and achieve global optimality. The previous observations hint that trajectories from our extremal family are excellent candidates for the time-optimal trajectory for their respective maneuvers.

IV. Conclusions

Analysis of the wrong-direction roll phenomenon in an extremal family for time-optimal roll around the velocity-vector maneuvers reveals how several levels of dynamic coupling, within the combat-aircraft dynamic model, correlate simultaneously to affect time optimality. In particular, the interaction between the kinematic and angular-rate dynamic subsystems is understood, and the extremal-family characteristic motion is revealed to be a hybrid one, a blend of two peculiar motion models that compromises the need for kinematic motion and the available control power. The tools and methodology used can be applied in analysis of more complex tactical-maneuvering problems.

Acknowledgments

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